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Reg. No. : .....

Name : .....

Fifth Semester B.Sc. Degree Examination, December 2019

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1543 : DIFFERENTIAL EQUATIONS

(2014 Admission onwards)

Time : 3 Hours Max. Marks : 80

PART – A

All the first **ten** questions are compulsory. They carry one mark each :

1. Find the integrating factor of $y'-(\sin x)y=4$.
2. Give an example of a differential equation of order 5.
3. Write down the standard form of a linear differential equation of first order.
4. Define doubling time.
5. State the fundamental theorem for homogeneous linear differential equation.
6. State the Existence and Uniqueness Theorem for initial value problem.
7. Define Wronskian.

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8. Show that the differential equation $(1+4xy+2y^2)\,dx+(1+4xy+2x^2)\,dy=0$ is exact.

9. Write down the auxiliary equation of the differential equation $y''-10y'+5y=0$.

10. Write down the standard form of Euler-Cauchy equation.

PART – B

Answer **any eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Solve the differential equation $y'-y=e^{2x}$.

12. Solve the differential equation $y'+3y=0$.

13. Solve the differential equation $y'+y\tan x=\cos^3 x$.

14. If 100 grams of radioactive carbon-14 are stored in a cave for 1000 years, how many grams will be left at that time?

15. Verify that e^x is an integrating factor of the differential equation $\sin y\,dx+\cos y\,dy=0$.

16. Solve the differential equation $2xy\,dx+x^2\,dy=0$.

17. Find a general solution of $y''+9y'+20y=0$.

18. Find a real general solution of $25y''+40y'+16+y=0$.

19. Write down the auxiliary equation of the differential equation $x^2y''+xy'+3y=0$.

20. Show that $\cos x$ and $\sin x$ are linearly independent.

21. Find a general solution of $x^2y''-20y=0$.

22. Explain Modification Rule for the Method of Undetermined Coefficients.

PART – C

Answer **any six** questions from among the question 23 to 31. These questions carry **4** marks each.

23. Solve the initial value problem $y' - xy = x;\ y(0) = 3$.

24. Find a curve in the xy - plane that passes through $(0,3)$ and whose tangent line at a point (x, y) has slope $\frac{2x}{y^2}$.

25. Solve the initial value problem $xy' - y = x;\ y(1) = 2$.

26. Find an integrating factor and solve the differential equation $2x \tan y\, dx + \sec^2 y\, dy = 0$.

27. Solve the initial value problem $y'' + y' - 2y = 0; y(0) = 4; y'(0) = -5$.

28. Using the method of reduction of order solve the differential equation $x^2 y'' - xy' + y = 0$, given that $y_1 = x$ is a solution.

29. Solve the boundary value problem $y'' + 4y = 0; y(0) = 3; y\left(\frac{\pi}{2}\right) = -3$.

30. Solve the initial value problem $4x^2 y'' + 24xy' + 25y = 0; y(1) 2; y'(1) = -6$.

31. Verify $y_p = e^{-3x}$ is a solution of $y'' - y = 8e^{-3x}$ and find the general solution.

PART – D

Answer **any two** questions from among the question 32 to 35. These questions carry **15** marks each.

32. (a) Solve the initial value problem $y'' - y = 0;\ y(0) = 4; y'(0) = -2$.

    (b) Use Euler's Method with a step size of 0.1 to make a table of approximate values of the solution of the initial value problem $y' = y - x;\ y(0) = 2$, over the interval $0 \le x \le 1$.

33. (a) According to United Nations data, the world population in 1998 was approximately 5.9 billion and growing at a rate of about 1.33% per year. Assuming an exponential growth model, estimate the world population at the beginning of the year 2023.

(b) Solve the initial value problem $y' + 4xy^2 = 0;\ y(0) = 1$.

34. Solve the initial value problem $y'' + 2y' + y = e^{-x};\ y(0) = -1; y'(0) = 1$, using the method of undetermined coefficients.

35. (a) Solve the differential equation $y'' + 9y = \sec 3x$, using the Method of Variation of parameters.

(b) Solve the initial value problem $y'' - 25y = 0;\ y(0) = 0;\ y'(0) = 20$.